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Numerical Study Of Finite Animal Poulation With Fertile And Non- Fertile Age Based Bifurcation

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Abstract
In this paper a mathematical model has been constructed for animal population of finite size living in abounded domain like reserved forest. The population has been divided in fertile and non-fertile classes .This model has been obtained in term of finite difference equation. The birth rate is assumed to be linear and death rate proportional to be square of the size. Numerical calculations have been worked out for some specific cases.
Keywords: Fertile and non-fertile, death rate, animal population.

1- Introduction

In this chapter we consider medium finite size species of finite population which are divided in two age groups. The growth and decrease the population depended on, births, deaths, migration and transition of the population. In this work in first age group we have taken linear births and deaths rate square of finite size and in the non fertile group while second in linear deaths rate of the finite size of population are taken. We worked out some numerical examples with growth and decay graph–pattern. The methodology is used application of difference equation .in this work.

2- Formulation of the Problem

Age based population P_n may be divided in to two parts

i.e. $$P_n = P_n^{(1)} + P_n^{(2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

$P_n^{(1)}$ = Population of infants juveniles (pre fertile).

$P_n^{(2)}$ = Population of the fertile age group.

The change in Population of first age group occurs due to births, migration, and transition from one age group to second age group and quadratic deaths.

Change the Population $P_n^{(1)}$ is given by

$$\Delta P_n^{(1)} = P_n^{(1)} - P_{n-1}^{(1)}$$

Therefore

$$\Delta P_n^{(1)} = B_1 P_{n-1}^{(2)} - D_1 (P_{n-1}^{(1)})^2 - T_1 (P_{n-1}^{(1)}) + \alpha P_{n-1}^{(1)} \ \ldots \ \ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

Where,

B_1 = Birth rate (uniform throughout)

D_1 =Quadratic death rate of n^{th} generation

T_1 = Transition rate from $P_{n-1}^{(1)}$ to $P_{n-1}^{(2)}$ in n^{th} generation in one group

$\alpha P_{n-1}^{(1)}$ =Mmigration rate of one age group and α is fixed ratio of one age group of n^{th} generation and α is strictly less than 1.

From equation (1) and (2).we obtain.

$$P_n^{(1)} = (1 - T_1 + \alpha)P_{n-1}^{(1)} + B_1 P_{n-1}^{(2)} - D_1 (P_{n-1}^{(1)})^2$$

Let

$$(1 - T_1 + \alpha) = X_n$$

Than we get

$$P_n^{(1)} = X_n P_{n-1}^{(1)} + B_1 P_{n-1}^{(2)} - D_1 \left(P_{n-1}^{(1)}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Similarly, we can find the difference equation of second group.

$$\Delta P_n^{(2)} = P_n^{(2)} - P_{n-1}^{(2)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This gives

$$\Delta P_n^{(2)} = T_2 P_{n-1}^{(1)} - D_2 P_{n-1}^{(2)} + \beta P_{n-1}^{(2)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where,

$T_2 =$ Transition rate from $P_{n-1}^{(1)}$ to $P_{n-1}^{(2)}$ in n^{th} generation in second group

$D_2 =$ Death rate of n^{th} generation

$\beta P_{n-1}^{(2)}$ =Migration rate of second age group and β is fixed ratio of second age group of n^{th} generation and β is strictly less than 1.

From equations (4) and (5).we get

$$P_n^{(2)} = (1 - D_2 + \beta)P_{n-1}^{(2)} + T_2 P_{n-1}^{(1)}$$

Let $T_2 P_{n-1}^{(1)} = \mu T_2 P_{n-1}^{(2)}$

[Where μ is prescribed ratio and (0< μ<1)]

Then

$$P_n^{(2)} = (1 - D_2 + \beta)P_{n-1}^{(2)} + \mu T_2 P_{n-1}^{(2)}$$

Or

$$P_n^{(2)} = (1 - D_2 + \beta + \mu T_2)P_{n-1}^{(2)}$$

$$P_n^{(2)} = Y_n P_{n-1}^{(2)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where

$$Y_n = (1 - D_2 + \beta + \mu T_2)$$

3- Solution of Difference Equation

Solution of equation (6) is given by V. P.Saxena (2011)

$$P_n^{(2)} = \sum_{i=1}^{n} Y_i P_0^{(2)}$$

Replacing n by $(n-1)$

$$P_{n-1}^{(2)} = \sum_{i=1}^{n-1} Y_i P_0^{(2)}$$

Putting the value of $P_{n-1}^{(2)}$ in equation (3)

$$P_n^{(1)} = X_n P_{n-1}^{(1)} + B_1 P_{n-1}^{(2)} - D_1 \left(P_{n-1}^{(1)}\right)^2$$

$$P_n^{(1)} = X_n P_{n-1}^{(1)} + B \left[\sum_{i=1}^{n-1} Y_i P_0^{(2)}\right] - D_1 \left(P_{n-1}^{(1)}\right)^2$$

$$P_n^{(1)} = \sum_{i=1}^{n} X_i P_0^{(1)} + B \sum_{i=1}^{n} \prod_{i=1}^{n} X_i \prod_{i=1}^{n-1} Y_i P_0^{(2)} - D (P_0^{(1)})^2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Special Case-1

$$Y_1 = Y_2 = \cdots \dots \dots \dots \dots \dots \dots \dots \dots \dots Y_n = Y$$

$$T_1 = T_2 = \cdots \dots \dots \dots \dots \dots \dots \dots \dots \dots T_n = T$$
$$D_1 = D_2 = \cdots \dots \dots \dots \dots \dots \dots \dots \dots D_n = D$$

Then
$P_n^{(2)} = Y^n P_0^{(2)}$

Step-1: if n=1
$P_1^{(2)} = Y P_0^{(2)}$

Step-2, if n=2
$P_2^{(2)} = Y^2 P_0^{(2)}$

Step-3, if n=3
$P_3^{(2)} = Y^3 P_0^{(2)}$

Step-4, if n=4
$P_4^{(2)} = Y^4 P_0^{(2)}$

Step-5, if n=5
$$P_5^{(2)} = Y^5 P_0^{(2)}$$

Step-6, if n=6
$$P_6^{(2)} = Y^6 P_0^{(2)}$$

Numerical Examples

Numerical calculations have been carried out using Mat lab 6.5 programming and also graphs are drawn.

Example-1:

(a) Let $P_0^{(1)} = 1000$, $P_0^{(2)} = 2000$

(b) Let $P_0^{(1)} = 1500$, $P_0^{(2)} = 3000$

Then
$$P_1^{(2)} = Y P_0^{(2)}$$
$$P_1^{(2)} = 2000Y$$

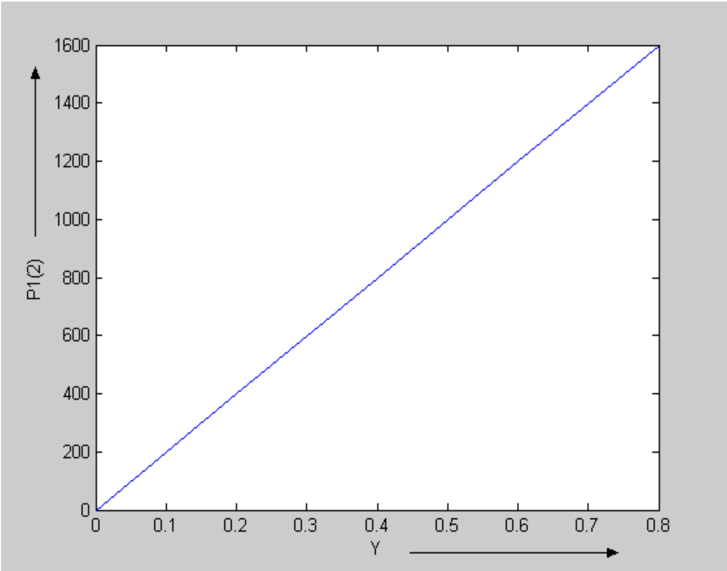


Figure 1 (a) Graph between Y and $P_1^{(2)}$

$$P_1^{(2)} = YP_0^{(2)}$$
$$P_1^{(2)} = 3000Y$$

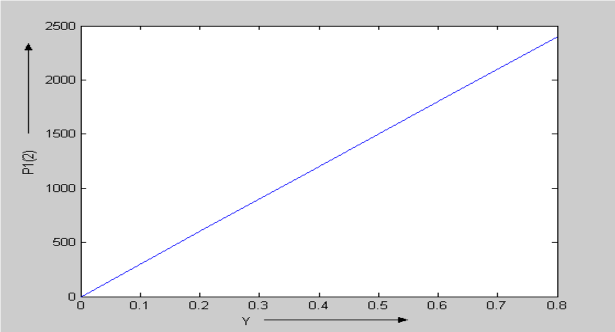


Figure 1 (b) Graph between Y and $P_1^{(2)}$

Special Case-2

Let

$$X_1 = X_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = X_n = X$$
$$T_1 = T_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = T_n = T$$
$$D_1 = D_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = D_n = D$$

Then $$P_n^{(1)} = X^n \; P_{n-1}^{(1)} + BP_{n-1}^{(2)} - D\left(P_{n-1}^{(1)}\right)^2$$

Step-1, if n=1

$$P_1^{(1)} = XP_0^{(1)} + BP_0^{(2)} - D\left(P_0^{(1)}\right)^2$$

Step-2, if n=2

$$P_2^{(1)} = X^2 \; P_1^{(1)} + B(X+Y)P_0^{(2)} - D\left(P_1^{(1)}\right)^2$$

Step-3, if n=3

$$P_3^{(1)} = X^3 \; P_0^{(1)} + B(X^2 + XY + Y^2)P_0^{(2)} - D\left(P_0^{(1)}\right)^2$$

Step-\4, if n=4

$$P_4^{(1)} = X^4 \; P_0^{(1)} + B(X^3 + X^2Y + XY^2 + Y^3)P_0^{(2)} - D\left(P_0^{(1)}\right)^2$$

Step-5, if n=5

$$P_5^{(1)} = X^5 \; P_0^{(1)} + B(X^4 + X^3Y + X^2Y^2 + XY^3 + Y^4)P_0^{(2)} - D\left(P_0^{(1)}\right)^2$$

Step-6, if n=6

$$P_6^{(1)} = X^6 \; P_0^{(1)} + B(X^5 + X^4Y + X^3Y^2 + X^2Y^3 + XY^4 + Y^5)P_0^{(2)} - D\left(P_0^{(1)}\right)^2$$

Numerical Examples:

Example-2

Numerical calculations have been carried out using Mat lab 6.5 programming and also graph.

(a) Let $P_0^{(1)} = 1000$, $P_0^{(2)} = 2000$, B=0.15, D=0.60

(b) Let $P_0^{(1)} = 1500$, $P_0^{(2)} = 3000$, B=0.15, D=0.60

Then

$$P_1^{(1)} = XP_0^{(1)} + BP_0^{(2)} - D\left(P_0^{(1)}\right)^2$$
$$P_1^{(1)} = 1000X + 300 - 0.6(1000000)$$

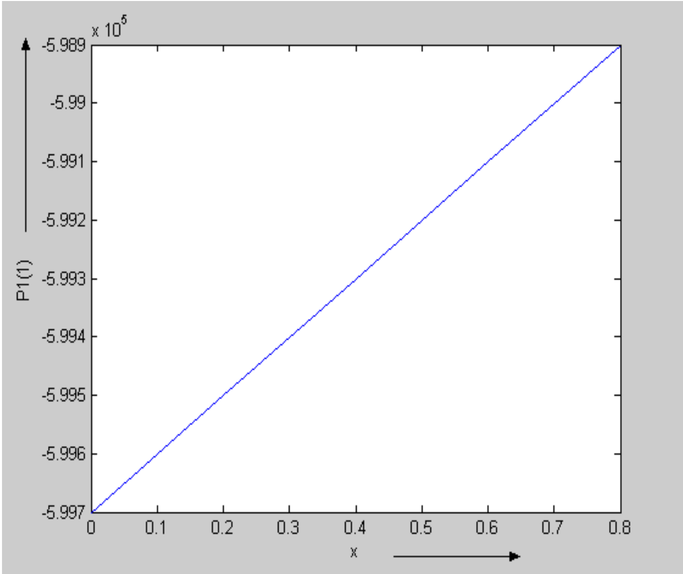


Figure 2 (a) Graph between X and $P_1^{(1)}$

$$P_1^{(1)} = XP_0^{(1)} + BP_0^{(2)} - D\left(P_0^{(1)}\right)^2$$

$$P_1^{(1)} = 1500X + 450 - 0.6(2250000)$$

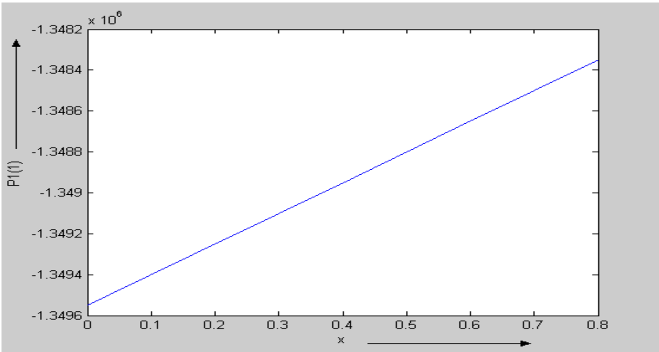


Figure 2 (b) Graph between x and $P_1^{(1)}$

Example-3

(a) Let $P_0^{(1)} = 1000$, $P_0^{(2)} = 2000$, B=0.15, D=0.60

(b) Let $P_0^{(1)} = 1500$, $P_0^{(2)} = 3000$, B=0.15, D=0.60

Then

$$P_2^{(1)} = X^2 \; P_1^{(1)} + B(X+Y)P_0^{(2)} - D\left(P_1^{(1)}\right)^2$$

$$P_2^{(1)} = 1000X^2 + 300(X+Y) - 0.6(1000000)$$

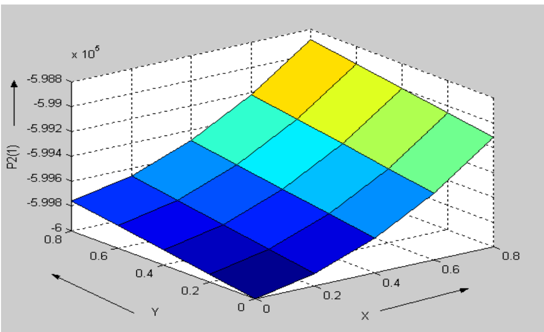
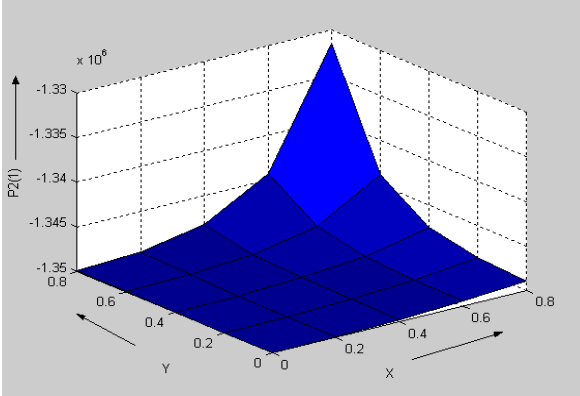


Figure 3 (a) Graph between X, Y and $P_2^{(1)}$

$$P_2^{(1)} = X^2 \; P_1^{(1)} + B(X+Y)P_0^{(2)} - D\left(P_1^{(1)}\right)^2$$
$$P_2^{(1)} = 1500X^2 + 450(X+Y) - 0.6(2250000)$$



tween X,Y and $P_2^{(1)}$

Figure 3 (b) Graph between X, Y and $P_2^{(1)}$

4- Conclusion

In this research paper, the graphs between the parameters X and y and the various populations are increasing with intrinsic growth in the non fertile age based population and decreasing in fertile age based population. This model mostly protect the finite population of medium size of species in reserve forest.

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